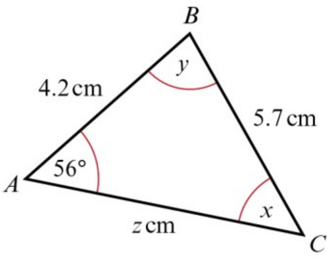
Exercise 6E

1 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin x}{4.2} = \frac{\sin 56°}{5.7}$$

$$\sin x = \frac{4.2\sin 56°}{5.7}$$

$$x = \sin^{-1}\left(\frac{4.2\sin 56°}{5.7}\right)$$

$$= 37.65...°$$

$$x = 37.7° (3 \text{ s.f.})$$

$$\text{So } y = 180° - (56° + 37.7°)$$

$$= 86.3°$$

$$y = 86.3° (3 \text{ s.f.})$$

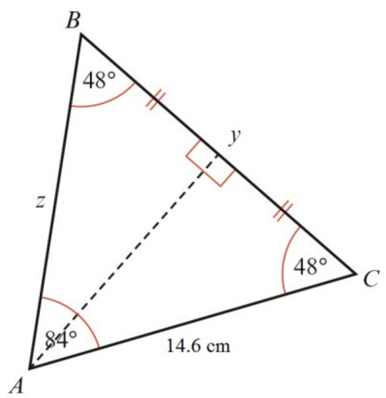
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{z}{\sin y} = \frac{5.7}{\sin 56°}$$

$$\text{So } z = \frac{5.7\sin y}{\sin 56°}$$

$$= 6.86° (3 \text{ s.f.})$$

b $x = 180° – (48 + 84)°$
$x = 48°$



As angle B = angle C, z = 14.6 cm.

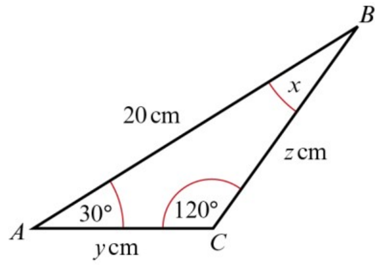
b Using the line of symmetry through A:

$$\cos 48° = \frac{\frac{y}{2}}{14.6}$$

$$\text{So } y = 29.2\cos 48°$$

$$= 19.5 \text{ cm} (3 \text{ s.f.})$$

c



$$x = 180° - (120° + 30°)$$

$$= 30°$$

Using the line of symmetry through C:

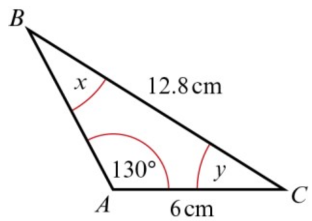
$$\cos 30° = \frac{10}{y}$$

$$\text{So } y = \frac{10}{\cos 30°}$$

$$= 11.5 \text{ cm} (3 \text{ s.f.})$$

Since ΔABC is isosceles with $AC = CB$,
$z = 11.5 \text{ cm} (3 \text{ s.f.})$

d



Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\frac{\sin 130°}{12.8} = \frac{\sin x}{6}$$

$$\text{So } \sin x = \frac{6\sin 130°}{12.8}$$

$$= 0.359\,08\ldots$$

$$\Rightarrow x = 21.0° (3 \text{ s.f.})$$

$$\text{So } y = 180° - (130° + x)$$

$$= 28.956...°$$

1 d $\Rightarrow y = 29.0° \text{ (3 s.f.)}$

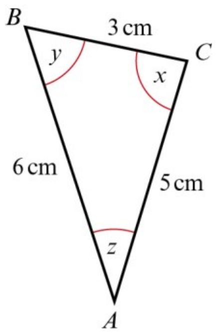
Using $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$$\frac{z}{\sin y} = \frac{12.8}{\sin 130°}$$

So $z = \dfrac{12.8 \sin y}{\sin 130°}$

$= 8.09 \text{ cm (3 s.f.)}$

e



Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$$\cos x = \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5}$$

$$= -0.0\dot{6}$$

$$x = 93.8° \text{ (3 s.f.)}$$

Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$$\frac{\sin y}{5} = \frac{\sin x}{6}$$

$$\sin y = \frac{5 \sin x}{6}$$

$$y = \sin^{-1}\left(\frac{5 \sin x}{6}\right)$$

$$= 56.25\ldots°$$

$$y = 56.3° \text{ (3 s.f.)}$$

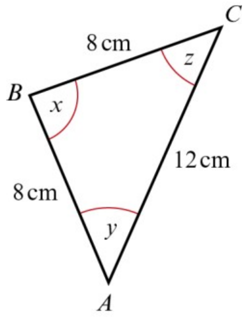
Using the angle sum for a triangle:

$z = 180° - (x + y)°$

$= 29.926\ldots°$

$z = 29.9° \text{ (3 s.f.)}$

f



Using the line of symmetry through B:

$\cos y = \frac{6}{8}$

$= \frac{3}{4}$

$y = \cos^{-1}\left(\frac{3}{4}\right)$

$= 41.40\ldots°$

$y = 41.4° \text{ (3 s.f.)}$

As the triangle is isosceles:

$z = y$

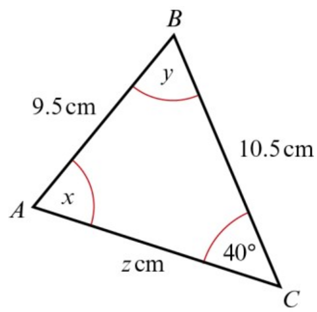
$= 41.4° \text{ (3 s.f.)}$

So $x = 180° - (y + z)°$

$= 97.2°$

$x = 97.2° \text{ (3 s.f.)}$

g



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin x}{10.5} = \frac{\sin 40°}{9.5}$$

$$\sin x = \frac{10.5 \sin 40°}{9.5}$$

$$x = \sin^{-1}\left(\frac{10.5 \sin 40°}{9.5}\right) \text{ or}$$

$$x = 180° - \sin^{-1}\left(\frac{10.5 \sin 40°}{9.5}\right)$$

$x = 45.27°$ or $x = 134.728\ldots°$

$x = 45.3° \text{ (3 s.f.)}$ or $x = 135° \text{ (3 s.f.)}$

1 g Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\frac{z}{\sin y} = \frac{9.5}{\sin 40°}$$

$$z = \frac{9.5 \sin y}{\sin 40°}$$

When $x = 45.3°$

$$y = 180° - (40 + 45.3)°$$
$$= 94.7°$$

So $y = 94.7$ (3 s.f.)

$$z = \frac{9.5 \sin y}{\sin 40°}$$
$$= 14.7 \text{ cm (3 s.f.)}$$

When $x = 134.728\ldots°$

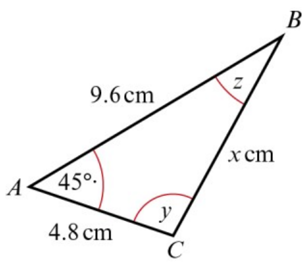
$$y = 180° - (40 + 134.72\ldots$$
$$= 5.27°$$

So $y = 5.27°$ (3 s.f.)

$$z = \frac{9.5 \sin y}{\sin 40°}$$
$$= 1.36 \text{ cm (3 s.f.)}$$

So $x = 45.3°,\ y = 94.7°,\ z = 14.7$ cm
or $x = 135°,\ y = 5.27°,\ z = 1.36$ cm

h



Using $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 4.8^2 + 9.6^2 - 2 \times 4.8 \times 9.6 \times \cos 45°$$
$$= 50.03\ldots$$
$$x = 7.07 \text{ cm (3 s.f.)}$$

Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$$\frac{\sin y}{9.6} = \frac{\sin 45°}{x}$$

$$\sin y = \frac{9.6 \sin 45°}{x}$$

$$y = \sin^{-1}\left(\frac{9.6 \sin 45°}{x}\right)$$

h $y = 73.68\ldots$

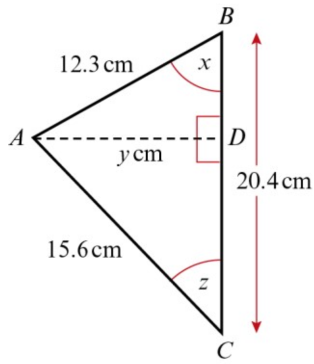
$y = 73.7°$ (3 s.f.)

Then

$$z = 180° - (45 + 73.68\ldots$$
$$= 61.32\ldots°$$
$$z = 61.3° \text{ (3 s.f.)}$$

So $x = 7.07$ cm, $y = 73.7°$, $z = 61.3°$

i



Using $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$$\cos x = \frac{20.4^2 + 12.3^2 - 15.6^2}{2 \times 20.4 \times 12.3} \frac{1}{2}$$
$$= 0.6458\ldots$$
$$x = 49.77\ldots°$$
$$x = 49.8° \text{ (3 s.f.)}$$

In right-angled triangle *ABD*:

$$\sin x = \frac{y}{12.3}$$

So $y = 12.3 \sin x$

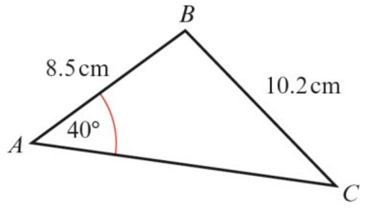
$$= 9.39 \text{ cm (3 s.f.)}$$

In right-angled triangle *ACD*:

$$\sin z = \frac{y}{15.6}$$
$$= 0.601\,99\ldots$$
$$z = 37.01\ldots°$$
$$z = 37.0° \text{ (3 s.f.)}$$

So $x = 49.8°,\ y = 9.39$ cm, $z = 37.0°$

2 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{8.5} = \frac{\sin 40°}{10.2}$

$\sin C = \frac{8.5 \sin 40°}{10.2}$

$C = \sin^{-1}\left(\frac{8.5 \sin 40°}{10.2}\right)$

$= 32.388...°$

$= 32.4° \text{ (3 s.f.)}$

$B = 180° - (40 + C)°$

$= 107.6...°$

$B = 108° \text{ (3 s.f.)}$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{10.2 \sin B}{\sin 40°}$

$= 15.1 \text{ cm (3 s.f.)}$

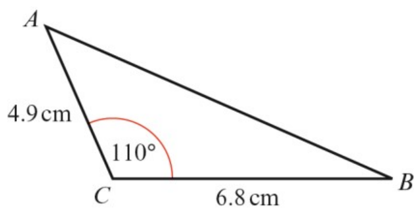
Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 10.2 \times 8.5 \times \sin 108°$

$= 41.228$

$= 41.2 \text{ cm}^2 \text{ (3 s.f.)}$

b



Using $c^2 = a^2 + b^2 - 2ab\cos C$

$AB^2 = 6.8^2 + 4.9^2 - 2 \times 6.8 \times 4.9 \times \cos 110°$

$= 93.04\ldots$

$AB = 9.6458\ldots$

$= 9.65 \text{ cm (3 s.f.)}$

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

2 b $\sin A = \frac{6.8 \sin 110°}{AB}$

$= 0.662\,45\ldots$

$A = 41.49°$

$= 41.5° \text{ (3 s.f.)}$

So $B = 180° - (110 + A)°$

$= 28.5° \text{ (3 s.f.)}$

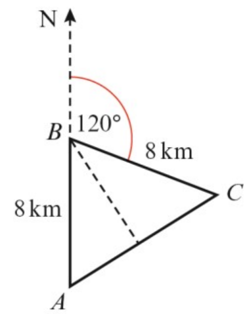
Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 6.8 \times 4.9 \times \sin 110°$

$= 15.655\ldots$

$= 15.7 \text{ cm}^2 \text{ (3 s.f.)}$

3



a Angle $ABC = 180° - 120°$

$= 60°$

As $\angle A = \angle C$, all angles are 60°.

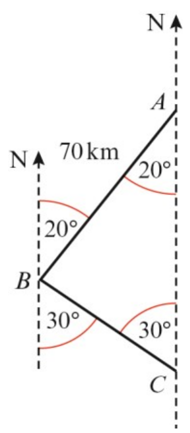
It is an equilateral triangle.

So $AC = 8$ km.

b As $\angle BAC = 60°$,

the bearing of C from A is 060°.

4



From the diagram

$\angle ABC = 180° - (20 + 30)°$

$= 130°$

4 Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\frac{AC}{\sin 130°} = \frac{70}{\sin 30°}$$

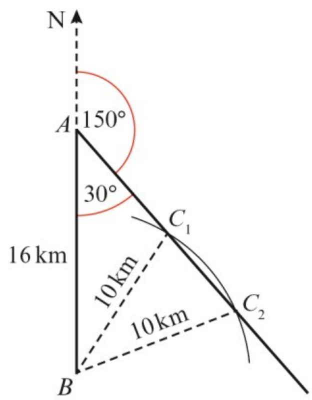
$$AC = \frac{70 \sin 130°}{\sin 30°}$$

$$= 107.246\ldots$$

$$AC = 107 \text{ km } (3 \text{ s.f.})$$

From the diagram, the bearing of C from A is 180˚.

5



Using the sine rule

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{16} = \frac{\sin 30°}{10}$$

$$\sin C = \frac{16 \sin 30°}{10}$$

$$= 0.8$$

$$C = \sin^{-1}(0.8) \text{ or } C = 180° - \sin^{-1}(0.8)$$

$$C = 53.1° \text{ or } C = 126.9°$$

$$\angle AC_2B = 53.1°, \angle AC_1B = 127° \; (3 \text{ s.f.})$$

(Store the correct values; these are not required answers.)

Triangle BC_1C_2 is isosceles, so C_1C_2 can be found using this triangle, without finding AC_1 and AC_2.

Use the line of symmetry through B:

$$\cos \angle C_1C_2B = \frac{\frac{1}{2}C_1C_2}{10}$$

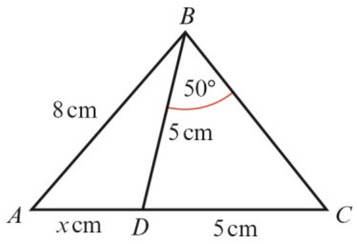
$$\Rightarrow C_1C_2 = 20 \cos \angle C_1C_2B$$

$$= 20 \cos \angle AC_2B$$

$$= 20 \cos 53.1°$$

$$\Rightarrow C_1C_2 = 12 \text{ km}$$

6 a



In the isosceles ΔBDC:

$$\angle BDC = 180° - (50 + 50)°$$

$$= 80°$$

$$\text{So } \angle BDA = 180° - 80°$$

$$= 100°$$

Using the sine rule in ΔABD

$$\frac{\sin A}{a} = \frac{\sin D}{d}$$

$$\Rightarrow \frac{\sin A}{5} = \frac{\sin 100°}{8}$$

$$\Rightarrow \sin A = \frac{5 \sin 100°}{8}$$

$$\text{So } A = \sin^{-1}\left(\frac{5 \sin 100°}{8}\right)$$

$$= 37.9886\ldots$$

$$\angle ABD = 180° - (100 + A)°$$

$$= 42.01\ldots°$$

Using $\dfrac{b}{\sin B} = \dfrac{d}{\sin D}$

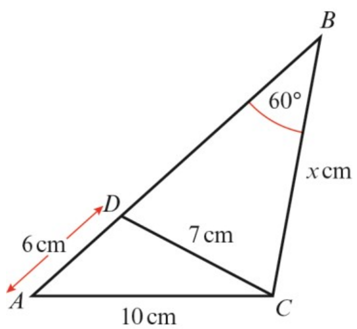
$$\frac{x}{\sin B} = \frac{8}{\sin 100°}$$

$$x = \frac{8 \sin B}{\sin 100°}$$

$$= 5.436\ldots$$

$$x = 5.44 \text{ cm } (3 \text{ s.f.})$$

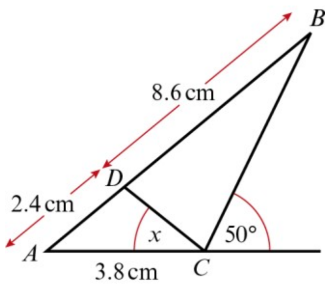
b



In ΔADC, using $\cos A = \dfrac{c^2 + d^2 - a^2}{2cd}$

6 b $\cos A = \dfrac{6^2 + 10^2 - 7^2}{2 \times 6 \times 10}$
$= 0.725$
So $A = 43.53...°$
Using the sine rule in ΔABC:
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$
So $\dfrac{x}{\sin A} = \dfrac{10}{\sin 60°}$
$\Rightarrow x = \dfrac{10\sin A}{\sin 60°}$
So $x = 7.95$ cm (3 s.f.)

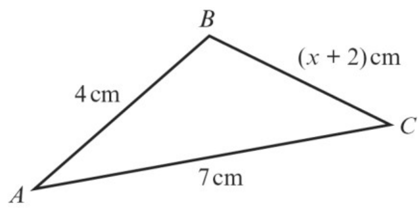
c



In ΔABC, $c = 11$ cm, $b = 3.8$ cm,
$\angle ACB = 130°$, $(180° - 50°)$
Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$
$\sin B = \dfrac{3.8\sin 130°}{11}$
$= 0.2646\ldots$
$B = 15.345...°$
So $A = 180° - (130 + B)°$
$= 34.654...°$

In ΔADC, $c = 2.4$ cm, $d = 3.8$ cm,
$A = 34.654...°$
Using the cosine rule:
$a^2 = c^2 + d^2 - 2cd\cos A$
So $DC^2 = 2.4^2 + 3.8^2 - 2 \times 2.4 \times 3.8 \times \cos A$
$= 5.1959\ldots$
$\Rightarrow DC = 2.279\ldots$
Using the sine rule:
$\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$
$\sin x = \dfrac{2.4\sin A}{DC}$
$= 0.598\,69\ldots$
$x = 36.8$ (3 s.f.)

7



a As $AB + BC > AC$
$4 + (x + 2) > 7$
$\Rightarrow x + 2 > 3$
$\Rightarrow x > 1$
As $AB + AC > BC$
$4 + 7 > x + 2$
$\Rightarrow 9 > x$
So $1 < x < 9$

b Using $b^2 = a^2 + c^2 - 2ac\cos B$
i $7^2 = (x+2)^2 + 4^2 - 2(x+2) \times 4 \times \cos 60°$
$49 = x^2 + 4x + 4 + 16 - 4(x+2)$
$49 = x^2 + 4x + 4 + 16 - 4x - 8$
So $x^2 = 37$
$\Rightarrow x = 6.08$ cm (3 s.f.)
Area $= \frac{1}{2}ac\sin B$
$= \frac{1}{2} \times 8.08 \times 4 \times \sin 60°$
$= 13.9949\ldots$
$= 14.0$ cm^2 (3 s.f.)

ii $7^2 = (x + 2)^2 + 4^2$
$- 2 \times (x + 2) \times 4 \times \cos 45°$
$49 = x^2 + 4x + 4 + 16$
$-(8\cos 45°)x - 16\cos 45°$
So:
$x^2 + (4 - 8\cos 45°)x$
$-(29 + 16\cos 45°) = 0$
or $x^2 + 4(1 - \sqrt{2})x$
$-(29 + 8\sqrt{2}) = 0$

Use the quadratic formula
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1$
$b = 4 - 8\cos 45°$
$= 4(1 - \sqrt{2})$
$= -1.6568\ldots$

7 b ii $c = -(29 + 16\cos 45°)$

$= -(29 + 8\sqrt{2})$

$= -40.313\ldots$

$x = 7.23\text{ cm } (3\text{ s.f.})$

(The other value of x is less than –2.)

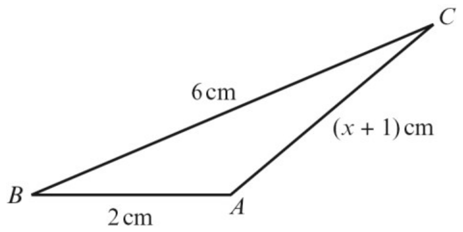
Area $= \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 4 \times 9.23 \times \sin 45°$

$= 13.05\ldots$

$= 13.1\text{ cm}^2$ (3 s.f.)

8 a



Using $b^2 = a^2 + c^2 - 2ac\cos B$ where $\cos B = \frac{5}{8}$

$(x+1)^2 = 6^2 + 2^2 - 2 \times 6 \times 2 \times \frac{5}{8}$

$x^2 + 2x + 1 = 36 + 4 - 15$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

So $x = 4 (x > -1)$

b Use identity, $\cos^2 x + \sin^2 x = 1$.

$\cos B = \frac{5}{8}$

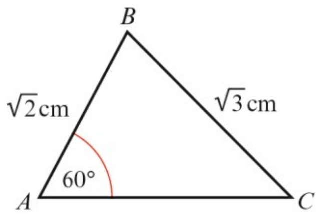
So $\sin \angle ABC = \frac{\sqrt{39}}{8}$

Area $= \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 6 \times 2 \times \frac{\sqrt{39}}{8}$

$= 4.68\text{ cm}^2$ (3 s.f.)

9



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\sin C = \frac{\sqrt{2}\sin 60°}{\sqrt{3}}$

$= 0.7071\ldots$

$C = \sin^{-1}\left(\frac{\sqrt{2}\sin 60°}{\sqrt{3}}\right)$

$= 45°$

$B = 180° - (60 + 45)°$

$= 75°$

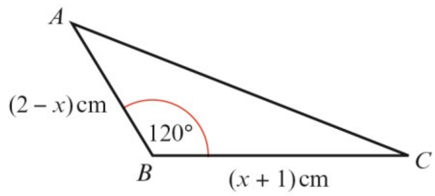
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{AC}{\sin 75°} = \frac{\sqrt{3}}{\sin 60°}$

So $AC = \frac{\sqrt{3}\sin 75°}{\sin 60°}$

$= 1.93\text{ cm } (3\text{ s.f.})$

10



a Using the cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos B$

$AC^2 = (x+1)^2 + (2-x)^2$

$- 2(x+1)(2-x)\cos 120°$

$= (x^2 + 2x + 1) + (4 - 4x + x^2)$

$+ (x+1)(2-x)$

$= x^2 + 2x + 1 + 4 - 4x + x^2$

$- x^2 + 2x - x + 2$

$= x^2 - x + 7$

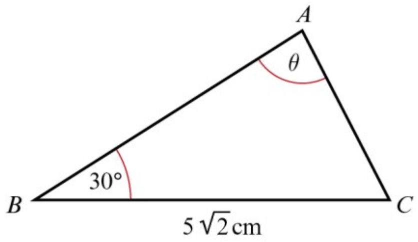
b Completing the square:

$x^2 - x + 7 \equiv \left(x - \frac{1}{2}\right)^2 + 7 - \frac{1}{4}$

$\equiv \left(x - \frac{1}{2}\right)^2 + 6\frac{3}{4}$

This is a minimum when $x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$.

11



Using $\frac{b}{\sin B}=\frac{a}{\sin A}$

$$\frac{AC}{\sin 30°}=\frac{5\sqrt{2}}{\sin\theta}$$

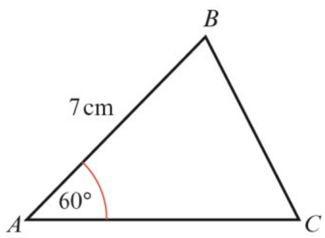
$$AC=\frac{5\sqrt{2}\sin 30°}{\left(\frac{\sqrt{5}}{8}\right)}$$

$$AC=\frac{5\sqrt{2}\sin 30°\times 8}{\sqrt{5}}$$

$$=\left(\sqrt{5}\sqrt{2}\right)\left(8\sin 30°\right)$$

$$=4\sqrt{10}\text{ cm}$$

12



Using the cosine rule:

$a^2=b^2+c^2-2bc\cos A$

with $a=x,\ b=(8-x),\ c=7$ and $A=60°$

$$x^2=(8-x)^2+49-2(8-x)\times 7\times\cos 60°$$

$$=64-16x+x^2+49-7(8-x)$$

$$=64-16x+x^2+49-56+7x$$

$\Rightarrow 9x=57$

$\Rightarrow x=\frac{57}{9}=\frac{19}{3}=6\frac{1}{3}$

So $BC=6\frac{1}{3}$ cm and

$$AC=\left(8-6\tfrac{1}{3}\right)\text{cm}$$

$$=1\tfrac{2}{3}\text{cm}$$

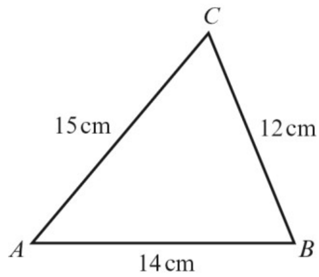
Area $=\frac{1}{2}bc\sin A$

$$=\tfrac{1}{2}\times 7\times\tfrac{5}{3}\times\sin 60°$$

$$=5.0518\ldots$$

$$=5.05\text{ cm}^2\ (3\text{ s.f.})$$

13 a



$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{12^2+15^2-14^2}{2(12)(15)}$$

$$=\frac{144+225-196}{360}$$

$C=61.278\ldots°=61.3°$ (3 s.f.)

b Use the formula.

Area $=\frac{1}{2}ab\sin C$

$$=\tfrac{1}{2}\times 12\times 15\times\sin 61.3°$$

$$=78.943\ldots$$

$$=78.9\text{ cm}^2\ (3\text{ s.f.})$$

14 a $\cos A=\frac{b^2+c^2-a^2}{2bc}$

$$\cos A=\frac{2.1^2+4.2^2-5.9^2}{2(2.1)(4.2)}$$

$$\cos A=\frac{4.41+17.64-34.81}{17.64}$$

$A=136.33\ldots°$

$\therefore$ Angle $DAB=136.3°$ (1 d.p.)

$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{3.5^2+7.5^2-5.9^2}{2(3.5)(7.5)}$$

$$=\frac{12.25+56.25-34.81}{52.5}$$

$C=50.080\ldots°$

$\therefore$ Angle $BCD=50.1°$

b Area ABD $=\frac{1}{2}bc\sin A$

$$=\tfrac{1}{2}\times 2.1\times 4.2\times\sin 136.3°$$

$$=3.046\ 79\ldots$$

14 b Area $BCD = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 3.5 \times 7.5 \times \sin 50.1°$

$= 10.069\ 04...$

Total area = 3.046 79 + 10.069 04

= 13.11583

∴ The area of the flower bed is 13.1 m^2.

c First find angle ADB:

$$\cos D = \frac{a^2 + b^2 - d^2}{2ab}$$
$$= \frac{5.9^2 + 2.1^2 - 4.2^2}{2(5.9)(2.1)}$$
$$= \frac{34.81 + 4.41 - 17.64}{24.78}$$

So $D = 29.440\ 849\ldots°$

Now find angle BDC:

$$\cos D = \frac{b^2 + c^2 - d^2}{2bc}$$
$$= \frac{3.5^2 + 5.9^2 - 7.5^2}{2(3.5)(5.9)}$$
$$= \frac{12.25 + 34.81 - 56.25}{41.3}$$
$$\cos D = \frac{3.5^2 + 5.9^2 - 7.5^2}{2(3.5)(5.9)}$$
$$\cos D = \frac{12.25 + 34.81 - 56.25}{41.3}$$

So $D = 102.856\ 97\ldots°$

Angle ADC = 29.440849 + 102.85697

= 132.298°

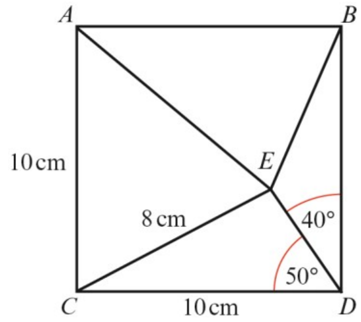
Now find the length AC:

$$d^2 = a^2 + c^2 - 2ac\cos D$$
$$= 3.5^2 + 2.1^2 - 2 \times 3.5 \times 2.1 \times \cos 132.298°$$
$$= 12.25 + 4.41 + 9.8929$$

So $d = 5.15$

The length of AC is 5.15 m.

15



Use the sine rule to work out angle CED.

$$\frac{\sin E}{e} = \frac{\sin D}{d}$$
$$\frac{\sin E}{10} = \frac{\sin 50°}{8}$$
$$\sin E = \frac{10\sin 50°}{8}$$

$E = 73.246\ 86°$ or $106.753\ 14°$

The angle is obtuse so

Angle CED = 106.753 14°

Angle ECD = 180° − 50° − 106.753 14°

= 23.25°

Use trigonometry to work out the height of triangle CDE.

$$\sin 23.25° = \frac{\text{height}}{8}$$

Height = 3.1575 cm

The height of triangle ABE = 10 − 3.1575

= 6.84 cm

Area of triangle = $\frac{1}{2} \times 10 \times 6.84 = 34.2$

∴ Area of the shaded triangle is 34.2 cm^2.